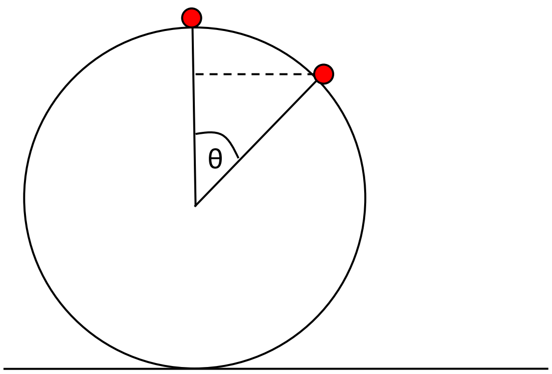
Teacher notes
Topic A

Ball falling off a sphere

A ball of mass *m* is placed at the top of a sphere of radius *R* and given the slightest push so that it moves away. At what angle θ does the ball lose contact with the sphere? Treat the "ball" as a point particle.



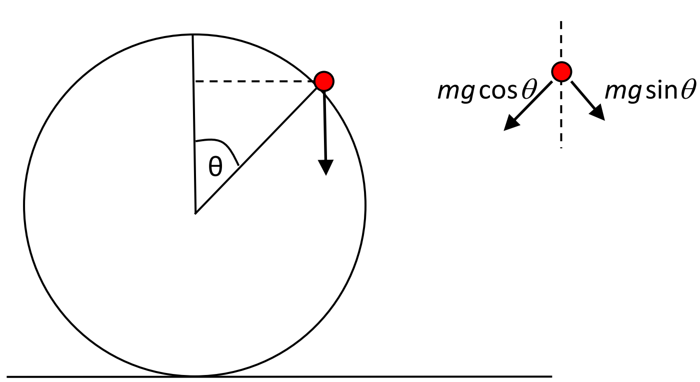
The sphere will fall a vertical distance $R - R\cos\theta$ and so the speed at the point shown is found by energy conservation to be:

$$0 + mgR(1-\cos\theta) = \frac{1}{2}mv^2 + 0$$

i.e.

$$v^2 = 2gR(1-\cos\theta)$$

When the ball is about to lose contact with the sphere the normal force from the sphere becomes zero and the only force on the ball is the weight. We take components of the weight along a radial axis and an axis tangent to the sphere:



We then have

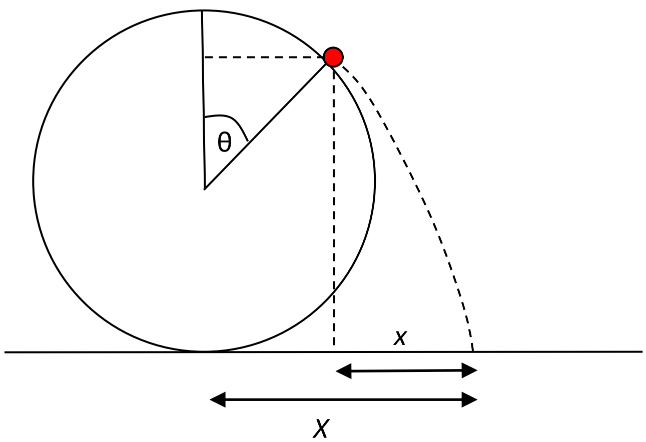
$$mg\cos\theta = m\frac{v^2}{R} = \frac{m}{R}\times 2gR(1-\cos\theta)$$

This gives

$$\cos\theta = 2(1-\cos\theta) \Rightarrow \cos\theta = \frac{2}{3}, \text{ so } \theta \approx 48.2^\circ.$$

It is interesting that the radius of the sphere has dropped out of the problem.

You may want to continue this problem by asking how far from the sphere the ball will land. (But this is beyond the difficulty of this course.)

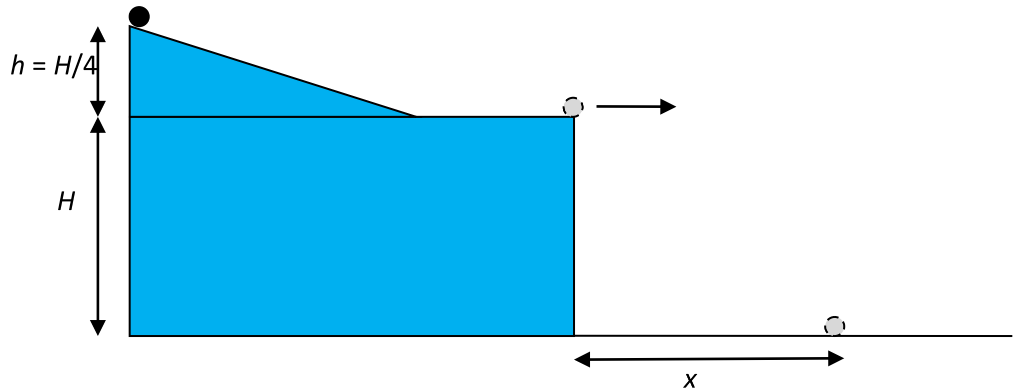


You should be able to show that $x = R\frac{\sqrt{800}-\sqrt{80}}{27} \approx 0.716R$. And hence $X = R\sin\theta + 0.716R \approx 1.46R$.

In the movie "The World is Not Enough" James Bond slides down what may be assumed to be a spherical dome but stays on it beyond the $\theta \approx 48.2^\circ$ angle; he is finally saved by grabbing on some ropes. How come Bond stays on the dome beyond $\theta \approx 48.2^\circ$?

An easier projectile motion problem:

A ball is released from the top of an inclined plane as shown. It lands on the ground a distance *x* from the block. What is *x*?



The ball will leave the rectangular block horizontally with speed $v=\sqrt{2gh}=\sqrt{2g\frac{H}{4}}=\sqrt{\frac{gH}{2}}$. The time to land is given by $H=\frac{1}{2}gt^2 \Rightarrow t=\sqrt{\frac{2H}{g}}$. Hence $x=\sqrt{\frac{gH}{2}}\sqrt{\frac{2H}{g}}=H$.